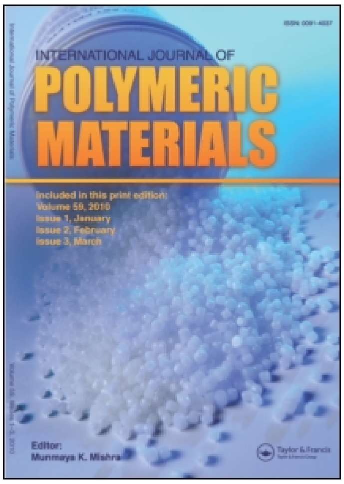


International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

Advanced Study on Non-Newtonian Flow of Polymer Melts in Extrusion Die of Plastic Net

Katsuhiko Ito[a]; Ikuro Arimatsu[a]
[a] Department of Mechanical Engineering, College of Engineering, Hosei Hosei University, Kajinocho, Koganei, Tokyo, Japan

To cite this Article Ito, Katsuhiko and Arimatsu, Ikuro(1984) 'Advanced Study on Non-Newtonian Flow of Polymer Melts in Extrusion Die of Plastic Net', International Journal of Polymeric Materials, 10: 4, 303 — 318
To link to this Article: DOI: 10.1080/00914038408078648
URL: http://dx.doi.org/10.1080/00914038408078648

Intern. J. Polymeric Mater., 1984, Vol. 10, pp. 303–318
0091–4037/84/1004–0303 $18.50/0

Advanced Study on Non-Newtonian Flow of Polymer Melts in Extrusion Die of Plastic Net†

KATSUHIKO ITO‡ and IKURO ARIMATSU

Department of Mechanical Engineering, College of Engineering, Hosei Hosei University, Kajinocho, Koganei, Tokyo, Japan

Broken section method is extended to analyze theoretically non-Newtonian flow of polymer melts with power law in the extrusion die of plastic net, in consideration of transversal drag action by die rotation. Die lip having overlap profile of semi-circle is broken into many sections, perpendicular to rotating boundary line. Computer approach is applied to analyze numerically the flow in each broken section. The flow isovels in discharge direction are indicated in various values of flow index and overlap ratio of upper and lower die lips. With increasing non-Newtonian behaviour, the profiles of the isovels in discharge direction are flattened and the area of the isovels of 0.9 increases to shift its location downwards. The flow isovels in rotating drag direction is plotted only in the case of half overlap ratio. With the decrease of flow index, the positive velocity profile in drag direction is flattened for larger area to cancel the sharp negative velocity near rotating drag boundary. Volumetric flow rate in various overlap ratios increases greatly with the decrease of flow index.

NOMENCLATURE

S = unit length of broken section (cm)
L = total width of die lips (cm)

† This paper is a sequel of "Broken Section Method for Analyzing Boundary Value Problems on Molten Flow in Polymer Processing—With Particular Emphasis on Extrusion Die," Report of the College of Engineering. Hosei university, No. 20 (March 1979) pp. 1–87.

‡ To whom all communications should be addressed.

N = number of sections (dimensionless)
H_i = separation of the die lips in the ith section (cm)
V = transversal drag velocity (cm/sec)
v_x = velocity component in x direction (cm/sec)
v_z = velocity component in z direction (cm sec)
n = flow index for power law fluids (dimensionless)
η = viscosity (poise)
p = pressure difference (kg/cm^2)
$\dot{\gamma}^0$ = standard state shear rate (sec^{-1})
η^0 = viscosity at standard state shear rate (poise)
y_1 = y defined by Eq. (7) (cm)
y_2 = y defined by Eq. (10) (cm)
Q_i = volumetric flow rate in the ith section (cm^3/sec)
Q_0 = overall volumetric flow rate (cm^3/sec)
R = radius of semi-circular die (cm)
$v_{z\,max}$ = overall maximum velocity component in z direction (cm/sec)
$v_{xp.max}$ = overall positive maximum velocity component in x direction (cm/sec)
y^*, y_1^*, y_2^*, v_x^*, v_z^*, G_x, G_z and η^* = dimensionless parameters, defined by Eq. (25)

1. INTRODUCTION

It is well known that Dr. F. B. Mercer (Netlon Ltd., England) invented his revolutional method for extrusion processing of plastic net through rotating die, of which the profile varies cyclically with time. In spite of a lot of applications of this method in plastic industries, however, there have been few researches on this interesting method of extrusion processing. Hence we reported some approaches[1,2] to theoretical analysis on melt processing of extrusion flow in plastic net by broken section[3] method.

However, effect of transversal drag action by die rotation on non-Newtonian flow behaviour was ignored in our previous[1] analysis. It is reported in this paper that broken section method is extended to analyze theoretically non-Newtonian flow of polymer melts in rotating extrusion die of plastic net, in consideration of transversal drag[2,4,5] action by die rotation.

2. BROKEN SECTION METHOD FOR ANALYZING TWO-DIMENSIONAL FLOW PROBLEM

First of all, it is assumed that flow is isothermal and laminar. No slip at all the die wall boundaries is also assumed. In addition, polymer melts are to be incompressive.

The non-Newtonian flow of polymer melts in extrusion die for plastic net is influenced by both pressure flow in extrusion direction and transversal drag flow induced by die rotation. The case where upper die is stationary but only lower die is rotated with linear velocity V (cm/sec), as shown in Figure 1 and Figure 2, is analyzed in this paper. Assuming that flow channel of extrusion die has a separation height sufficiently shallow compared with its width, as shown in Figure 2, the total width L (cm) of overlap die is broken into N sections, each of length S, where

$$S = L/N \tag{1}$$

The Cartesian coordinate with x, y and z axes is defined separately in each ith section, as shown in Figure 2 and Figure 3, taking transversal rotating direction as x-axis and extrusion flow direction as z-axis, respectively.

It is assumed that the non-Newtonian flow behaviour with power law (flow index: n) is applicable to analyze this complicated flow problem. Neglecting the flow in y-direction, the x and z components of the momentum equation in the ith section on this problem

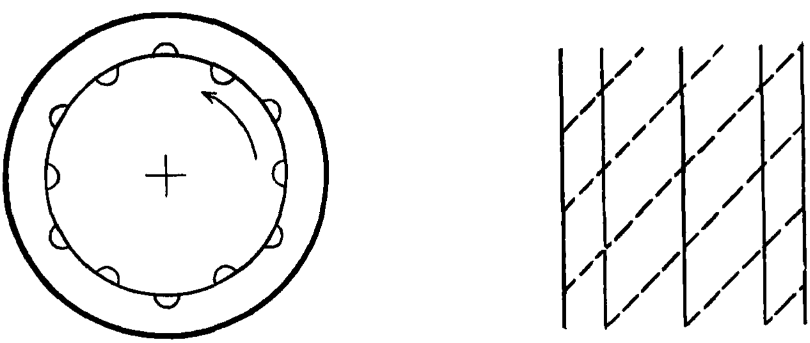


FIGURE 1 Rotating die for plastic net and extrudates.

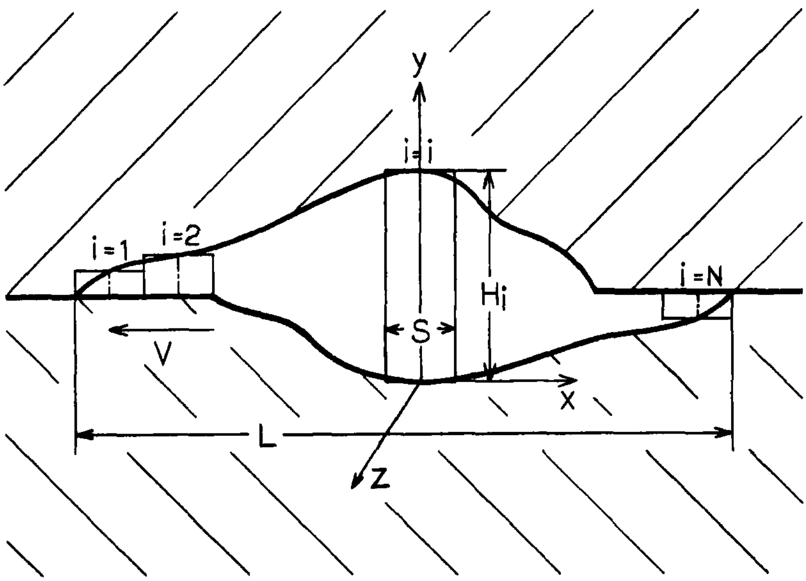


FIGURE 2 Broken sections of overlap die for plastic net.

are expressed in the following simultaneous differential equation.

$$\frac{\partial p}{\partial x}=\frac{d}{dy}\left(\eta\frac{dv_x}{dy}\right) \tag{2}$$

$$\frac{\partial p}{\partial z}=\frac{d}{dy}\left(\eta\frac{dv_z}{dy}\right) \tag{3}$$

$$\eta=\eta^0(\dot{\gamma})^{(1-n)}\left\{\left(\frac{dv_x}{dy}\right)^2+\left(\frac{dv_z}{dy}\right)^2\right\}^{(n-1/2)} \tag{4}$$

where p: pressure difference (kg/cm^2)
η: shear viscosity (poise)
v_x: velocity component in x direction (cm/sec)
v_z: velocity component in z direction (cm/sec)
n: flow index in power law fluid
$\dot{\gamma}^0$: standard shear rate ($\dot{\gamma}^0=1$ sec)
η^0: shear viscosity at standard shear rate (poise)

The boundary conditions for the above simultaneous differential equation are as follows.

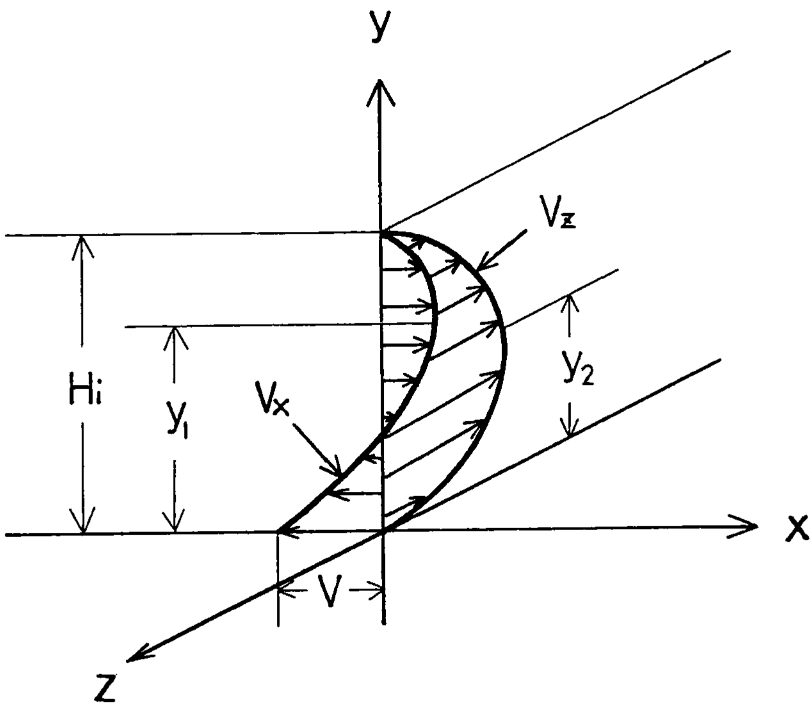


FIGURE 3 Velocity profile on the flow in extrusion die.

x direction:

$$v_x(y=0)=V \tag{5}$$

$$v_x(y=H_i)=0 \tag{6}$$

$$\left(\frac{dv_x}{dy}\right)_{y=y_1}=0 \tag{7}$$

z direction:

$$v_z(y=0)=0 \tag{8}$$

$$v_z(y=H_i)=0 \tag{9}$$

$$\left(\frac{dv_z}{dy}\right)_{y=y_2}=0 \tag{10}$$

where V: transversal drag velocity (cm/sec)
Q_i: volumetric flow rate in the ith section (cm^3/sec)
H_i: separation height of channel in the ith section (cm)
y_1: y defined by Eq. (7) (cm)
y_2: y defined by Eq. (10) (cm)

For the Newtonian case, it has been proved that y_1 and y_2 should be as follows, and hence the analysis can be performed easily.

$$y_1 = 2H_i/3 \tag{11}$$

$$y_2 = H_i/2 \tag{12}$$

From the power law Eq. (4) and the boundary conditions (7) and (10), the following results are obtained.

$$\frac{dv_x}{dy} = \frac{1}{\eta^0}(\dot{\gamma}^0)^{(n-1)}\left[\left(\frac{dv_x}{dy}\right)^2 + \left(\frac{dv_z}{dy}\right)^2\right]^{(1-n/2)} \cdot \left(\frac{\partial p}{\partial x}\right)(y - y_1) \tag{13}$$

$$\frac{dv_z}{dy} = \frac{1}{\eta^0}(\dot{\gamma}^0)^{(n-1)}\left[\left(\frac{dv_x}{dy}\right)^2 + \left(\frac{dv_z}{dy}\right)^2\right]^{(1-n/2)} \cdot \left(\frac{\partial p}{\partial z}\right)(y - y_2) \tag{14}$$

By solving simultaneously the above two equations for (dv_x/dy) and (dv_z/dy), the following differential equations are obtained.

$$\frac{dv_x}{dy} = \left\{\frac{1}{\eta^0(\dot{\gamma}^0)^{(1-n)}}\right\}^{(1/n)}\left(\frac{\partial p}{\partial x}\right)(y - y_1) \times\left[\left(\frac{\partial p}{\partial x}\right)^2 (y-y_1)^2 + \left(\frac{\partial p}{\partial z}\right)^2 (y-y_2)^2\right]^{(1-n/2n)} \tag{15}$$

$$\frac{dv_z}{dy} = \left\{\frac{1}{\eta^0(\dot{\gamma}^0)^{(1-n)}}\right\}^{(1/n)}\left(\frac{\partial p}{\partial z}\right)(y - y_2) \times\left[\left(\frac{\partial p}{\partial x}\right)^2 (y-y_1)^2 + \left(\frac{\partial p}{\partial z}\right)^2 (y-y_2)^2\right]^{(1-n/2n)} \tag{16}$$

Because of no flow discharge in the x-direction, the following condition should be satisfied.

$$\int_0^{H_i} v_x \, dy = 0 \tag{17}$$

We can convert the above differential equations and the boundary conditions into a dimensionless form, as follows.

$$\frac{dv_x^*}{d_y^*} = G_x(y^* - y_1^*)[G_x^2(y^* - y_1^*)^2 + G_z^2(y^* - y_2^*)^2]^{(1-n/2n)} \qquad (18)$$

$$\frac{dv_z^*}{dy^*} = G_z(y^* - y_2^*)[G_x^2(y^* - y_1^*)^2 + G_z^2(y^* - y_2^*)^2]^{(1-n/2n)} \qquad (19)$$

x direction:

$$v_x^*(y^* = 0) = -1 \qquad (20)$$

$$v_x^*(y^* = 1) = 0 \qquad (21)$$

$$\int_0^1 v_x^* \cdot dy^* = 0 \qquad (22)$$

z direction:

$$v_z^*(y^* = 0) = 0 \qquad (23)$$

$$v_z^*(y^* = 1) = 0 \qquad (24)$$

where

$$\left.\begin{array}{lll} y^* = y/H_i, & y_1^* = y_1/H_i, & y_2^* = y_2/H_i, \\ v_x^* = v_x/V, & v_z^* = v_z/V, & \\ G_x = \dfrac{\partial p}{\partial x} \cdot \dfrac{H_i^{(n+1)}}{\eta^* V^n}, & G_z = \dfrac{\partial p}{\partial z} \cdot \dfrac{H_i^{(n+1)}}{\eta^* V^n}, & \\ \eta^* = \eta^0(\dot{\gamma}^0)^{(1-n)}, & & \end{array}\right\} \qquad (25)$$

In the general numerical procedure, given G_z and n, we estimate G_x, y_1 and y_2, and obtain approximate values of velocities v_x and v_z. The estimated values are checked and, if different, the calculation is repeated with the improved estimates, as indicated in detail in the following section.

Thus, the volumetric flow rate Q_i in the ith section is

$$Q_i = SVH_i \int_0^1 v_z^* \, dy \qquad (26)$$

The overall flow rate Q_0 through die lip of plastic net is the total sum of the flow rate in each section.

$$Q_0 = \sum_{i=1}^{N} Q_i \qquad (27)$$

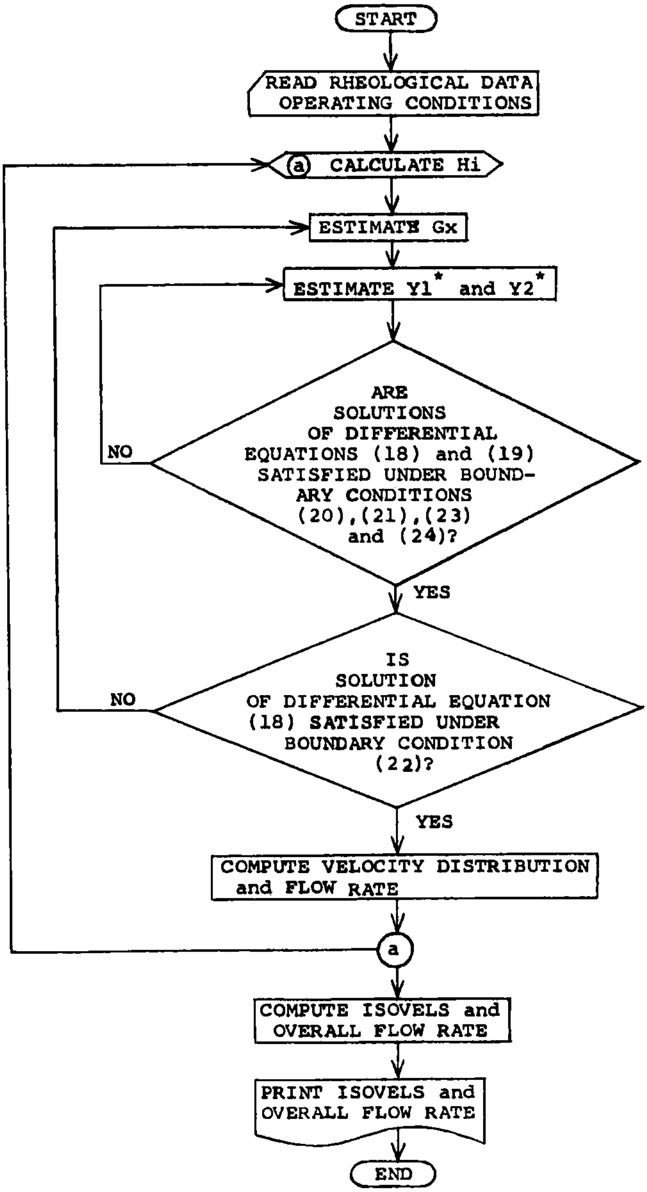


FIGURE 4 Computer chart.

3. COMPUTER CHART

The computer chart for numerical[6] analysis of this problem, is shown in Figure 4.

G_x, y_1^* and y_2^* are estimated and it is checked whether or not the numerical integrations obtained from the differential equations (18) and (19) satisfy all the boundary conditions (20), (21), (22), (23) and (24). If not, the calculation is repeated with the improved estimates. As indicated in the complicated computer chart of Figure 4, the calculation procedure is performed repeatedly for all the different values of H_i, until the satisfactory results of velocity profiles are obtained.

4. APPLICATION TO PLASTIC NET DIE OF SEMI-CIRCULAR PROFILE

Die lip having overlapped semi-circular cross-section, shown in Figure 5, is used for analyzing the molten flow in extrusion of plastic

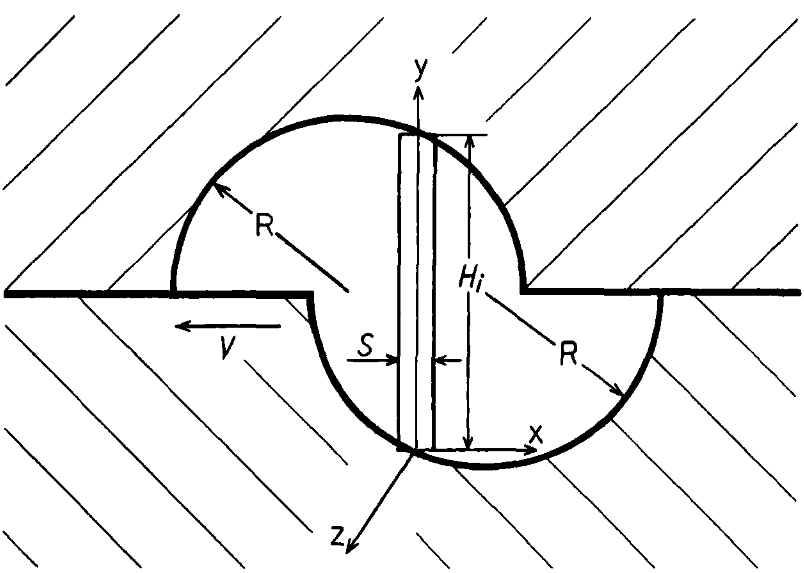


FIGURE 5 Broken sections of overlap die having semi-circular profile.

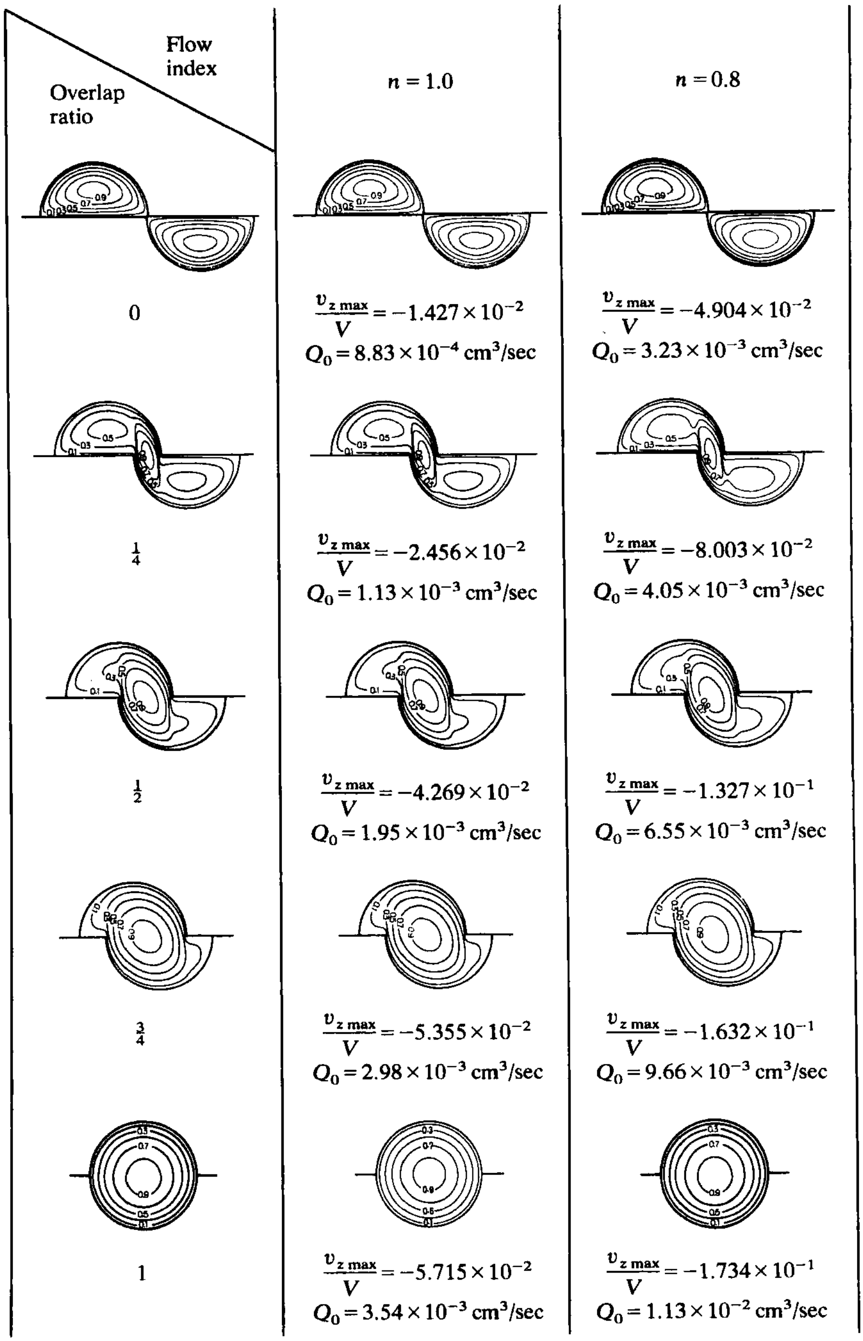
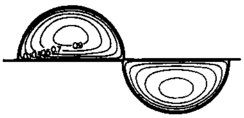
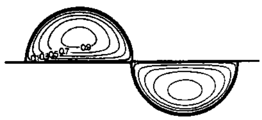
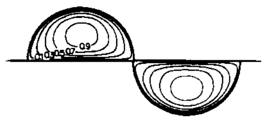
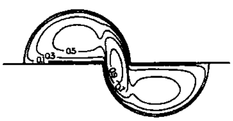
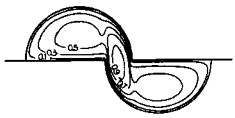
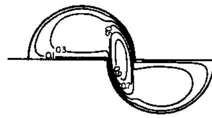
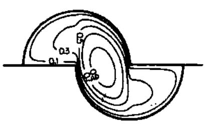
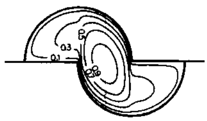
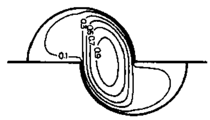
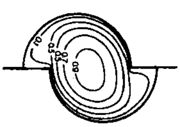
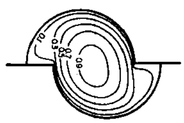
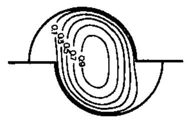
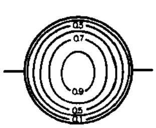
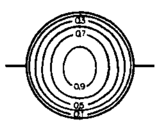
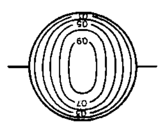


FIGURE 6 Isovels of v_z in the discharge direction and overall volumetric flow rate at various flow indices and overlap ratios.

Overlap ratio \ Flow index	$n = 0.6$	$n = 0.4$
 0	 $\frac{v_{z\,max}}{V} = -1.664 \times 10^{-1}$ $Q_0 = 1.15 \times 10^{-2}$ cm^3/sec	 $\frac{v_{z\,max}}{V} = -9.004 \times 10^{-1}$ $Q_0 = 6.17 \times 10^{-2}$ cm^3/sec
 $\frac{1}{4}$	 $\frac{v_{z\,max}}{V} = -2.708 \times 10^{-1}$ $Q_0 = 1.43 \times 10^{-2}$ cm^3/sec	 $\frac{v_{z\,max}}{V} = -1.833$ $Q_0 = 8.44 \times 10^{-2}$ cm^3/sec
 $\frac{1}{2}$	 $\frac{v_{z\,max}}{V} = -4.597 \times 10^{-1}$ $Q_0 = 2.33 \times 10^{-2}$ cm^3/sec	 $\frac{v_{z\,max}}{V} = -4.360$ $Q_0 = 1.96 \times 10^{-1}$ cm^3/sec
 $\frac{3}{4}$	 $\frac{v_{z\,max}}{V} = -5.768 \times 10^{-1}$ $Q_0 = 3.49 \times 10^{-2}$ cm^3/sec	 $\frac{v_{z\,max}}{V} = -6.381$ $Q_0 = 3.66 \times 10^{-1}$ cm^3/sec
 1	 $\frac{v_{z\,max}}{V} = -6.166 \times 10^{-1}$ $Q_0 = 4.11 \times 10^{-2}$ cm^3/sec	 $\frac{v_{z\,max}}{V} = -7.133$ $Q_0 = 4.54 \times 10^{-1}$ cm^3/sec

net. The die lip is broken into many sections of equal width $S = R/8$, where R: radius of semi-circle. The five different cases of overlap ratios of upper and lower die lips from the completely separated one to the completely overlapped one, are analyzed in this paper. In reality, the computation is performed in four different values of flow index of 1.0, 0.8, 0.6 and 0.4, using the following data.

$$\left.\begin{aligned} \dot{\gamma}^0 &= 1\ \mathrm{sec}^{-1} \\ \eta^0 &= 6.8\times 10^4\ \text{poise} \\ (\partial p/\partial z) &= 4.905\times 10^7\ \text{dyne}\backslash\text{cm}^2\ \text{cm} \\ V &= 15.7\ \text{cm/sec} \\ R &= 0.05\ \text{cm} \end{aligned}\right\} \qquad (28)$$

As shown in 20 illustrations of Figure 6, the results of v_z are plotted in the isovels with the ratio of v_z to the maximum velocity component $v_{z\,\max}$ in the section. The values of $v_{z\,\max}/V$ and Q_0 are additionally shown in Figure 6. On the other hand, the results of v_x are calculated only in the half overlap ratio and are shown in 4 illustrations of Figure 7, taking the ratio of v_x to the travelling velocity V of the lower semi-circular die, for various flow indices. Figure 7 contains also the values on the ratio of the positive maximum velocity component $v_{xp.\max}$ in the x-direction to V. The overall volumetric flow rates Q_0, in various overlap ratios of extrusion die of plastic net, are calculated from (26) and (27), and are plotted in Figure 8.

The resultant of v_z and v_x is the real velocity of polymer melts in extrusion die of plastic net. Due to closed channel, the flow in the x-direction causes only circulation, and so the v_x component should have plus and minus. Hence the resultant velocity vector of v_z and v_x is twisted.

The isovels of v_z for completely overlapped case (circular section) are perfect circles in the Newtonian case, due to no effect of transversal drag action in the x-direction on the flow in the z-direction. The $v_{z\,\max}$ increases significantly with the decrease of flow index, owing to the decrease of melt viscosity. The isovels $v_z\backslash v_{z\,\max}$, of which the value is close to 0.9, is located in the central zone of the overlapped dies. With the decrease of flow index, the velocity profiles are flattened and the area of the isovels $v_z\backslash v_{z\,\max} = 0.9$ increases, and further the location of the isovels $v_z\backslash v_{z\,\max} = 0.9$ goes

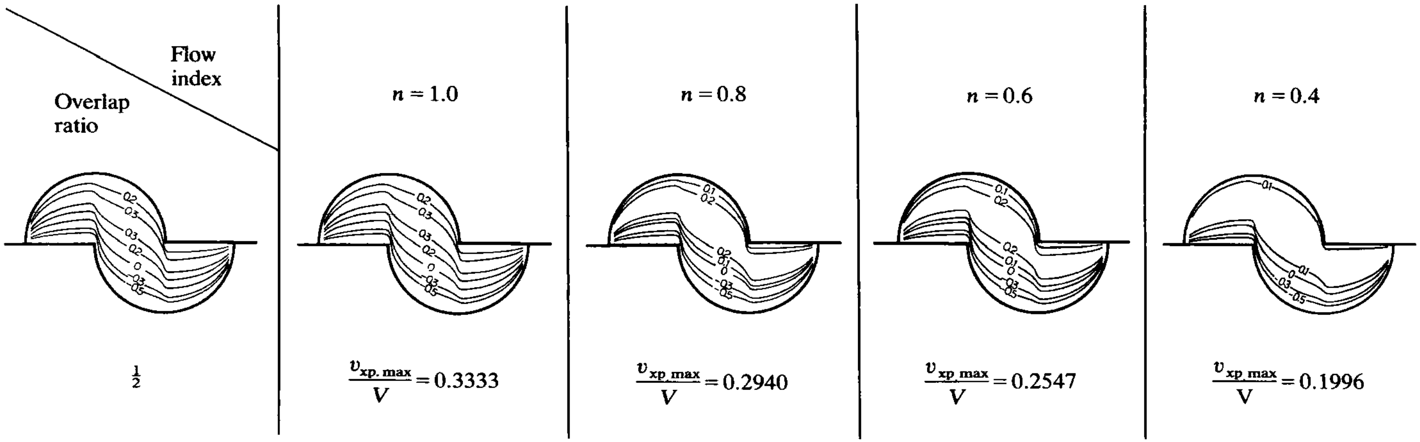


FIGURE 7 Isovels of v_x in the rotating drag direction at various flow indices, keeping overlap ratio constant 1 2.

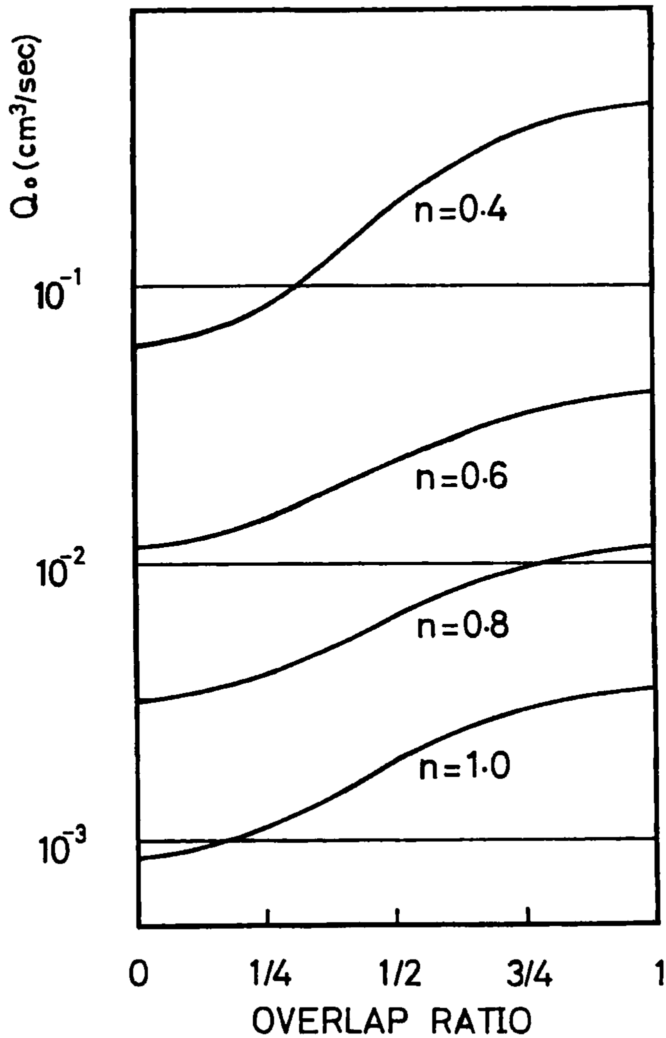


FIGURE 8 Relationship between overall volumetric flow rate and overlap ratio at various flow indices.

downwards more or less, due to the influence of the drag flow in the x-direction. It should be noted that, except in the Newtonian case, the isovels $v_z/v_{z\,max}$ are not exactly symmetrical about the center point of the cross-section.

The isovels v_x/V in the case of overlap ratio 1/2 are plotted in Figure 7, for various flow indices. With the decrease of flow index, the maximum of the positive v_x/V decreases and the velocity profile of the positive v_x/V is flattened for larger area, so that the small domain of the negative v_x/V should have sharp profile to cancel the positive volumetric flow rate and the isovels $v_x/V=-0.3$ and -0.5 are shifted downwards.

As shown in Figure 8, the overall volumetric flow rate Q_0 for various overlap ratios increases greatly with the decrease of flow index which causes the decrease of melt viscosity for power law fluid.

5. DISCUSSION

It is assumed in this analysis that volumetric flow rate in the x-direction is balanced in each ith section. In reality, however, the balance of volumetric flow rate in the x-direction should be established in the whole die section, considering that possible unbalance in each ith section may be existed.

It is noted that differential equation of the flow problem when upper and lower dies in plastic net processing are rotated each other in opposite direction, should be solved under the boundary conditions different from (20), (21), (22), (23) and (24). Further it should be considered in the coming analysis that inertia effect of unsteady discharge flow is existed by the fluctuation of volumetric flow rate which is caused by the variation of overlap ratio during die rotation.

Acknowledgement

This paper is one additional work of our numerous researches, entitled "Broken Section Method for Analyzing Boundary Value Problems on Molten Flow in Polymer Processing—With Particular Emphasis on Extrusion Die Design", which was originated with our first paper,[7] with Dean J. M. McKelvey, Washington University in St. Louis (Missouri, U.S.A.), during the appointment of one of the authors, Prof. K. Ito at Washington University in St. Louis for one year (1969–1970) under the auspices of the Senior Foreign Scientist Fellowship Program of the US National Science Foundation. The authors express their appreciation to Dean J. M. McKelvey for his initial cooperation of this problem. Further the authors are gratefully indebted to Prof. T. Hirano (College of Engineering, Hosei University, Tokyo, Japan) for his thoughtful suggestion on computer analysis in this study.

References

1. K. Ito, Y. Kato and S. Kimura, "Broken section method for analyzing molten flow in extrusion die of plastic net", *Rheology*, Vol. **3**: *Applications*, pp. 77–82, edited by G. Astaria, G. Marrucci and L. Nicolais, i.e. Proceedings of the 8th International Congress on Rheology (1980), published from the Plenum Publishing Corp., in 3 parts of which this is Vol. 3.
2. K. Ito, "Analysis on non-Newtonian flow of polymer melts in extrusion die of plastic Met" (Part 2), *Report on Progress in Polymer Physics in Japan*, **24**, pp. 113–116 (1981).
3. K. Ito, 'Broken section method for analyzing boundary value problems on molten flow in polymer processing—with particular emphasis on extrusion die design," *Report of the College of Engineering*, No. 20 (Hosei University, March 1979) pp. 1–87.
4. R. M. Griffith, "Fully developed flow in screw extruders," *I.E.C. Fundamentals*, **1** pp. 180–187 (1962).
5. Z. Tadmor and I. Klein, *Engineering principles of plasticating extrusion*, (Van Nostrand Reinhold Co., New York 1970), p. 274.
6. S. S. Kuo, *Computer applications of numerical methods*, (Addison-Wesley Publishing Co., Inc., 1972), Chapter 12.
7. J. M. McKelvey and K. Ito, "Uniformity of flow from sheeting dies," *Polymer Engineering and Science*, **11**, 3 pp. 258–263 (1971).